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Stagnation-point flow and heat transfer of a nanofluid towards stretching/shrinking cylinders

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Abstract - In this study we analyzed the stagnation point flow and heat transfer behavior of Ag-water nanofluid towards horizontal and exponentially permeable stretching/shrinking cylinders in presence of suction/injection. The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation which are then solved numerically using bvp4c technique with Matlab package. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables.

Index Terms - Stagnation-point flow, Stretching/shrinking cylinders, Suction/injection, Nanofluid, bvp4c, Volume fraction.

1 Introduction

The revolution of stagnation point flow has started by Massoudi and Ramezan [1]. It has various applications like transpiration, oil recovery, nuclear reactors and production. In this study they analyzed the heat transfer characteristics of a boundary layer flow of viscoelastic fluid towards a stagnation point. The stagnation point flow of two-dimensional steady nanofluid flow in the presence of stretching or shrinking sheet was analyzed by Bachok et al. [2]. Najib et al. [3] discussed the stagnation point flow over a stretching or shrinking cylinder with chemical reaction effect. In this study they highlighted the solutions for shrinking cylinder. Chen [4] discussed about laminar mixed convection flow through vertical continuously stretching sheet. Unsteady dusty gas flow in frenet frame was studied by Begewadi and Shantharajappa [5]. Numerical study on laminar boundary layer flow of a nanofluid was discussed by Khan and Pop [6]. Unsteady mixed convection flow by considering soret and Dufour effects was presented by Pal and Mandal [7]. Mohan Krishna et al. [8] are discussed the heat transfer characteristics of nanofluid by immersing the high conductivity nano materials in base fluids and they concluded that the effective thermal conductivity of the fluid increases appreciably and consequently enhances the heat transfer characteristics by suspending the high thermal conductivity of nano materials in to the base fluids. Ramana Reddy et al. [9] discussed radiation and chemical reaction effects on MHD dusty viscous flow by considering heat generation/absorption. Heat transfer characteristics of MHD viscoelastic fluid flow over nonlinear stretching sheet in the presence of radiation and heat generation/absorption was presented by Rafael Cortell [10]. Rana and Bhargava [11] presented finite element and finite difference methods for nonlinear stretching sheet problem Ece [12] proposed the similarity analysis for the laminar free convection boundary layer flow in the presence of a transverse magneticfield. Convective boundary layer flow of nanofluid past a linear stretching sheet was studied numerically by Makinde and Aziz [13]. Sandeep et al. [14] analyzed convective heat transfer of ethylene glycol based nanofluid. Mohan Krishna et al. [15] discussed radiation and chemical reaction effects on dusty viscous flow. Sandeep et al. [16] discussed MHD convective flow over a vertical plate by considering radiation, heat source and chemical reaction effects. Mustafaa et al. [17] boundary layer flow of a nano fluid over exponentially stretching sheet. A mathematical analysis to find the momentum and heat transfer characteristics of an incompressible, electrically conducting viscoelastic fluid over a linear stretching sheet was proposed by Abel et al. [18]. Very recently researchers [19-22] discussed the flow behavior of fluids at different channels.

In this study we are investigating the stagnation point flow and heat transfer behavior of Ag-water nanofluid towards horizontal and exponentially permeable stretching/shrinking cylinders with suction/injection. The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation which are then solved numerically. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables.

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2 MATHEMATICAL FORMULATION

Consider a steady stagnation-point flow and heat transfer of Ag-water nanofluid towards horizontal and exponential stretching/shrinking cylinders with radius R placed in an incompressible viscous nanofluid of constant temperature T_w. It is assumed that the free stream and stretching/shrinking velocities for horizontal and exponential cylinders are respectively $u_e = ax/L$, $u_w = cx/L$ and $u_e = ae^{x/L}$, $u_w = ce^{x/L}$, where a, c are constants and L is the characteristics length . A uniform heat source Q is considered in this study. The boundary layer equations as per above assumptions are given by

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0, \quad (1)$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+u_e\frac{\partial u_e}{\partial x}, \quad (2)$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \quad (3)$$

Where r is the coordinate measured in the radial direction, u and v are the velocity components in the x and r directions respectively. Further, T is the temperature in the boundary layer, ρ_{nf} and μ_{nf} are the density and the dynamic viscosity of the nano-fluid respectively, T_∞ is the free stream temperature, $(\rho c_p)_{nf}$ is the heat capacitance of nano-fluid, k_{nf} is the effective thermal conductivity of nanofluid.

Boundary conditions for horizontal cylinder

$$u=u_w \quad v=v_w, T=T_w, \text{ at } r=R$$
$$u\to u_e, T\to T_\infty \;\; as \;\; r\to\infty \quad (4)$$

Boundary conditions for exponential cylinder

$$u=u_w, v=v_w, T=T_w=T_\infty+T_0 e^{x/2L} \text{ at } r=R$$
$$u\to u_e, T\to T_\infty \;\; as \;\; r\to\infty \quad (5)$$

Where v_w is the suction $(v_w<0)$ or injection $(v_w>0)$ velocity. The nanofluid constants are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,$$
$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,$$
$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}, \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \quad (6)$$

Where ϕ is the volume fraction of the nano particles. The subscripts f and s refer to fluid and solid properties respectively.

For getting the similarity solutions of equations (1)-(3) with respect to the boundary conditions (4), we are setting the following similarity transformation

$$\eta=\frac{r^2-R^2}{2R}\sqrt{\frac{a}{\nu_f L}}, \psi=\sqrt{\frac{\nu_f a}{L}}xRf(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad (7)$$

Similarly, for obtaining the similarity solutions of equations (1)-(3) with respect to the boundary conditions (5), we are setting the following similarity transformation

$$\eta=\frac{r^2-R^2}{2R}\sqrt{\frac{a}{2\nu_f L}}e^{x/2L}, \psi=\sqrt{2\nu_f La}\,Rf(\eta)\,e^{x/2L},$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad (8)$$

Where $T-T_\infty=A(x/L)^2\theta(\eta)$, A is a positive constant, η is the similarity variable, ψ is the stream function defined as $u=r^{-1}\partial\psi/\partial r$ and $v=-r^{-1}\partial\psi/\partial x$, which is identically satisfied the continuity equation (1). By defining η in this form, the boundary conditions at $r=R$ reduce to the boundary conditions at $\eta=0$ which is more convenient for numerical computations.

Substituting (7) into equations (2) and (3) we get the following nonlinear ordinary differential equations for horizontal cylinder

$$\frac{1}{(1-\phi)^{2.5}}\left[(1+2\eta K)f'''+2Kf''\right]+\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)\left(ff''-f'^2\right)+1=0, \quad (9)$$

$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{\left(1-\phi+\phi\left((\rho c_p)_s/(\rho c_p)_f\right)\right)}\left[(1+2\eta K)\theta''+2K\theta'\right]+f\theta'-2f'\theta=0, \quad (10)$$

Substituting (8) into equations (2) and (3) we get the following nonlinear ordinary differential equations for exponential cylinder

$$\frac{1}{(1-\phi)^{2.5}}\left[(1+2\eta K)f'''+2Kf''\right]+\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)\left(ff''-2f'^2\right)+2=0, \quad (11)$$

$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{\left(1-\phi+\phi\left((\rho c_p)_s/(\rho c_p)_f\right)\right)}\left[(1+2\eta K)\theta''+2K\theta'\right]+ \quad (12)$$
$$(f\theta'-2f'\theta)=0,$$

Subject to the boundary conditions are:

$$f(0)=S,\ f'(0)=\lambda,\theta(0)=1,$$
$$f'(\infty)\to 0,\ \theta(\infty)\to 0, \quad (13)$$

Where K is the curvature parameter, $\Pr$ is the Prandtl number, Q_H is the heat source parameter, S is the suction/injection parameter, here $S>0$ for suction and $S<0$ for injection and λ is the stretching/shrinking parameter, here $\lambda>0$ for stretching and $\lambda<0$ for shrinking, these are given by

$$K=\left(\nu_f L/aR^2\right)^{1/2},\lambda=c/a,$$
$$\Pr=\nu_f/\alpha_f,S=-v_w\left(2v_f/u_w\right)^{1/2}, \quad (14)$$

The main physical quantities are interest of $f''(0)$ being a measure of the skin friction and the temperature gradient $-\theta'(0)$. Our aim is to find how the values of $f''(0)$ and $-\theta'(0)$ vary with the non-dimensional governing parameters for horizontal and exponential cylinders.

3 RESULTS AND DISCUSSION

Equations (9) to (12), subject to the boundary conditions (13) are solved numerically. For numerical results we considered $\Pr=6.2,\eta=30,\mathrm{K}=0.2,\ \lambda=2,$ and $S=1$. These values kept as common in entire study except the varied values as displayed in respective figures and tables. Results shows the influence of non-dimensional governing parameters like curvature parameter K, suction/injection parameter S,nano particles volume fractionϕ, and stretching/shrinking parameter λ on velocity and temperature profiles along with skin friction coefficient and Nusselt number. Figs. 1 and 2 show the effect of curvature parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is clear from figures that increases in curvature parameter enhance the velocity as well as temperature profiles. It is evident from figures that the curvature parameter showed mixed performance on both horizontal and exponential cylinders. Also, observed that Ag-water nanofluid is more influenced by curvature parameter on exponential cylinder.

Figs. 3 and 4 show the influence of volume fraction of nano particles on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is clear from figures that enhancement in volume fraction of nano particles decreases the velocity profiles and improves the temperature profiles of the flow. This is due to the fact that enhancement in volume fraction of nano particles reduces the velocity boundary layer thickness due to the friction near the walls. At the same time which helps to enhance the thermal conductivity of the flow. Figs. 5 and 6 depict the influence of suction/injection parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is observed from the figures that increase in suction/injection parameter decreases the velocity and temperature profiles of Ag-water nanofluid. Figs. 7 and 8 display the influence of stretching/shrinking parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is noticed from figures that the enhancement in stretching/shrinking parameter increases the velocity profiles and declines the temperature profiles in exponential cylinder but it shows reverse action in horizontal cylinder. Table 1 depicts the thermophysical properties of the base fluid (water) and the Silver nano particles. Table 2 shows the influence of non dimensional governing parameters on friction factor and Nusselt number. It is evident from the table that increase in curvature parameter increases the skin friction coefficient and reduces the Nusselt Number. Enhancement in volume fraction of nano particles depreciates the friction factor and heat transfer rate. In this case it is expected the increase in heat transfer rate but it may vary according to the remaining governing parameters we have chosen in present study. Friction factor and Sherwood number are not influenced by heat source parameter. But a raise in heat source parameter depreciates the Nusselt number. Enhance in suction/injection parameter, Stretching/shrinking parameter declines the friction factor and improves the heat transfer rate.

Table 1: Thermo physical properties of water and Ag -nano particles.

	$\rho(Kg\,m^{-3})$	$c_p(J\,Kg^{-1}\,K^{-1})$	$k(Wm^{-1}\,K^{-1})$
H_2O	997.1	4179	0.613

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Ag	10500	235	429

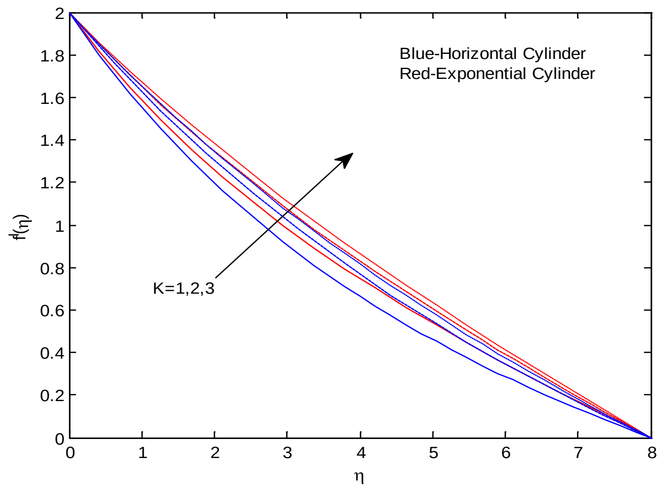

Figure 1: Velocity profiles for different values of curvature parameter K

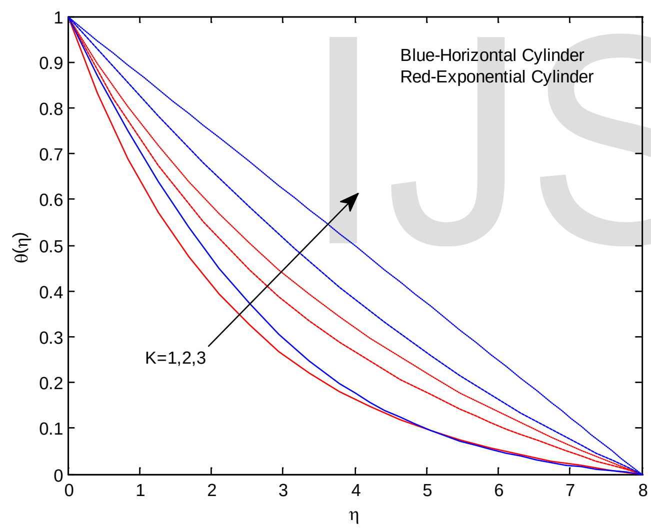

Figure 2: Temperature profiles for different values of curvature parameter K

Table 2: Variation in $f''(0)$ and $-\theta'(0)$ for different values of K, S, ϕ and λ.

K	ϕ	S	λ	$f''(0)$	$-\theta'(0)$
1	0.1	1	2	-0.497791	0.318779
2	0.1	1	2	-0.402994	0.178077
3	0.1	1	2	-0.366919	0.125130
0.2	0.1	1	2	-0.968429	1.031945
0.2	0.2	1	2	-1.071073	0.799302
0.2	0.3	1	2	-1.070482	0.620104
0.2	0.1	1	2	-0.968429	1.031945
0.2	0.1	2	2	-1.087543	1.121396
0.2	0.1	3	2	-1.218134	1.207558
0.2	0.1	1	-0.1	0.138950	0.213878
0.2	0.1	1	0	0.120915	0.246264
0.2	0.1	1	0.1	0.100540	0.276152

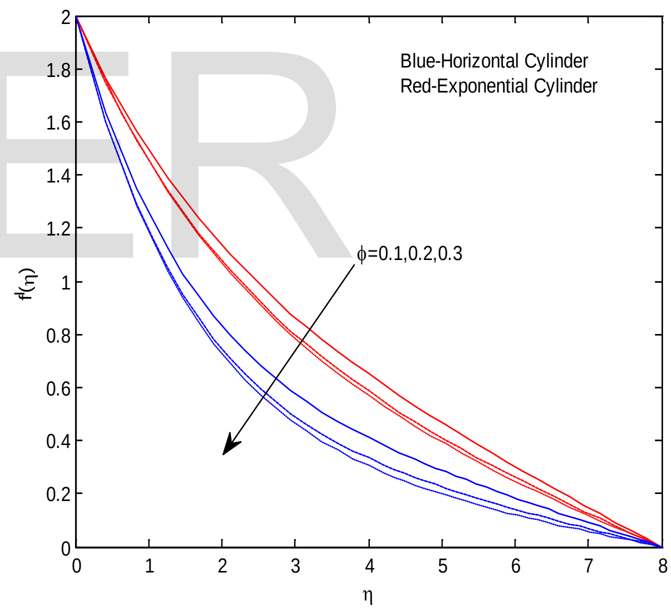

Figure 3: Velocity profiles for different values of nano particle volume fraction ϕ

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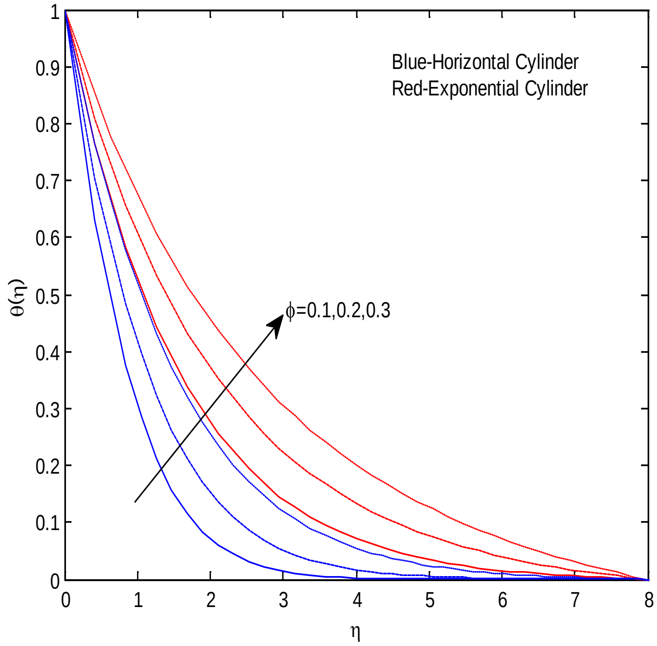

Figure 4: Temperature profiles for different values of nano particle volume fraction ϕ

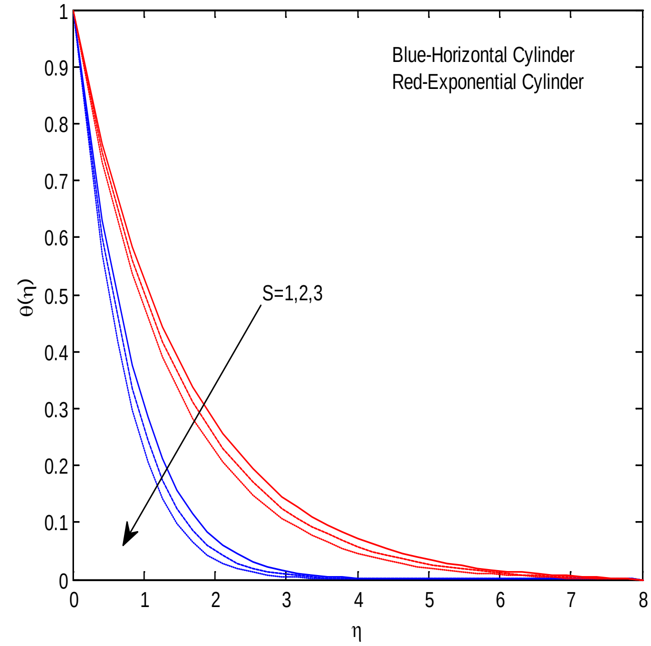

Figure 6: Temperature profiles for different values of suction/injection parameter S

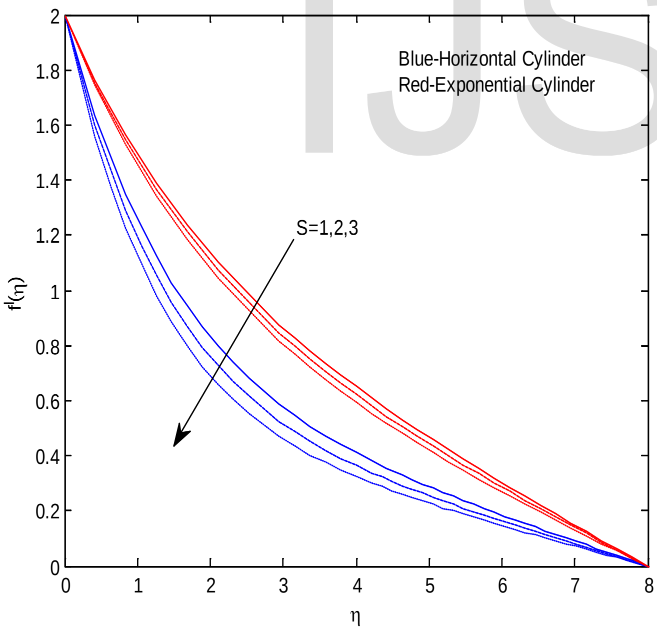

Figure 5: Velocity profiles for different values of suction/injection parameter S

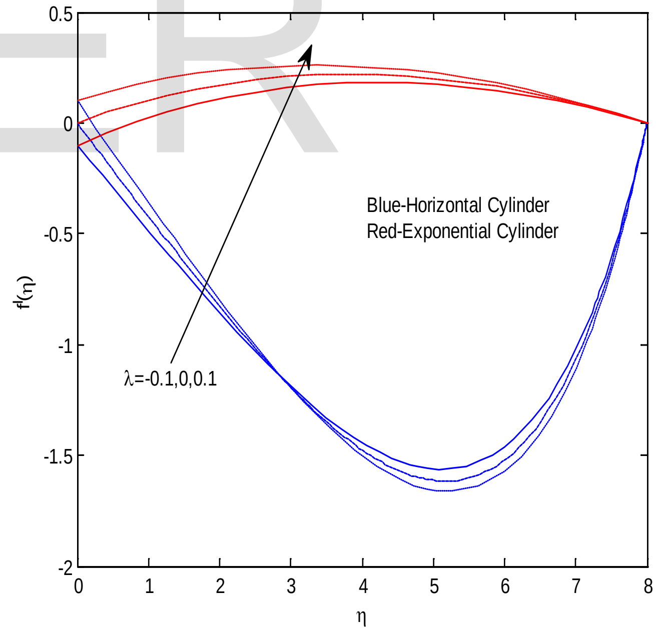

Figure 7: Velocity profiles for different values of stretching/shrinking parameter λ

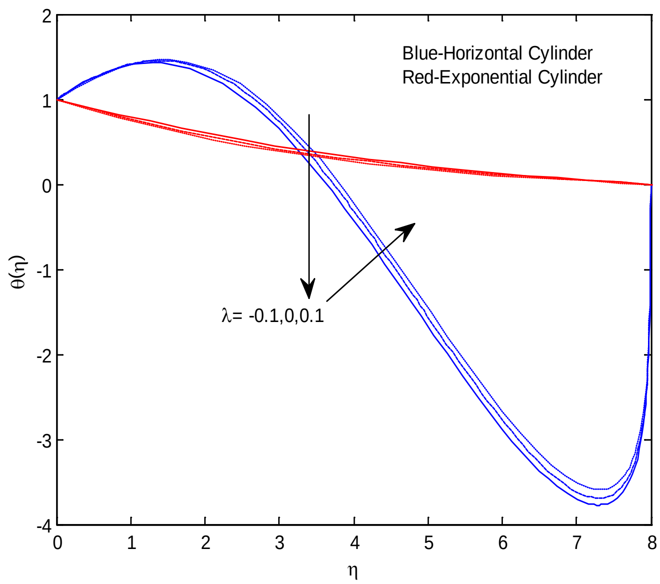

Figure 8: Temperature profiles for different values of stretching/shrinking parameter λ

4 CONCLUSIONS

In this study we are investigated the stagnation point flow and heat transfer behavior of Ag-water nanofluid towards horizontal and exponentially permeable stretching/shrinking cylinders with suction/injection . The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation which are then solved numerically. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables. The conclusions are made as follows:

- Curvature parameter have tendency to increase the friction factor and reduce the heat transfer rate.
- A rise in the nano particle volume fraction improves the temperature profiles of the flow.
- Increase in suction/injection parameter and stretching/shrinking parameters helps to enhance the heat transfer rate.
- Solutions exist only for certain range of stretching/shrinking parameter, for higher values of stretching/shrinking parameter velocity of the fluid is equal to free stream velocity.
- Flow and heat transfer behavior of Ag-water nanofluid through the horizontal and exponential stretching/shrinking cylinders are non-unique.

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